Cognitive bias approach to the acquisition of disjunction

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Inclusive and exclusive disjunction

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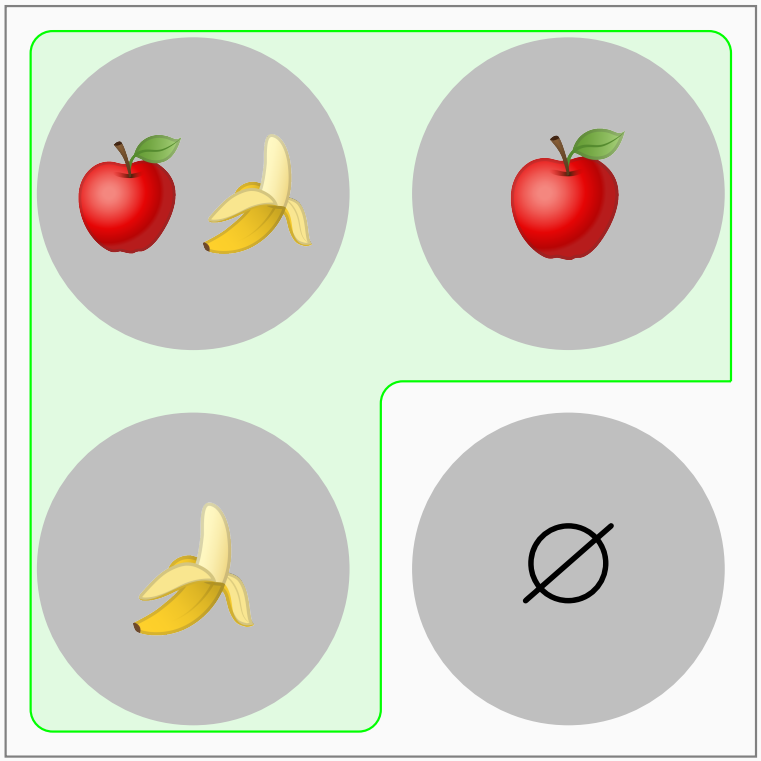

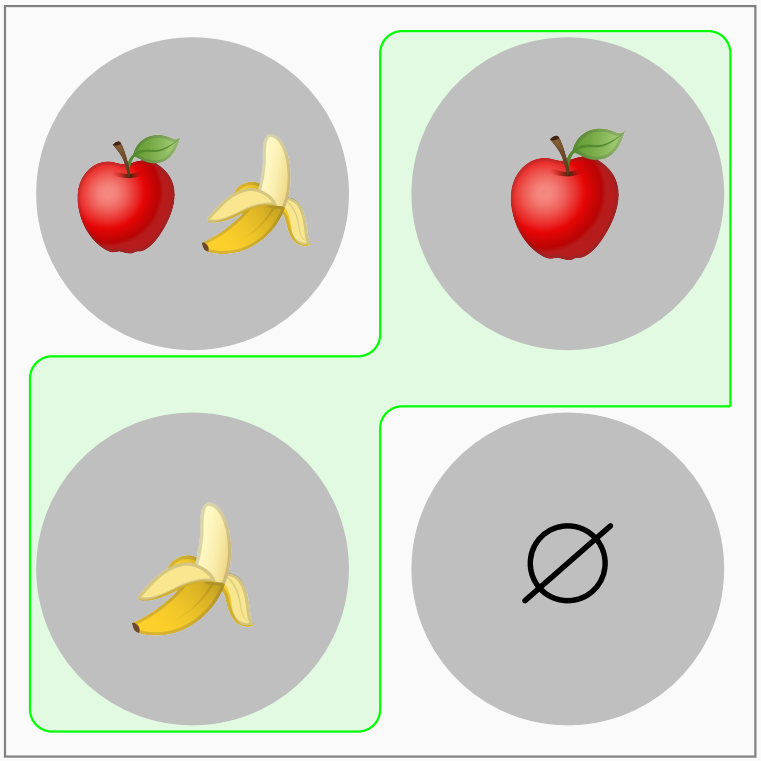

Deriving exclusive readings

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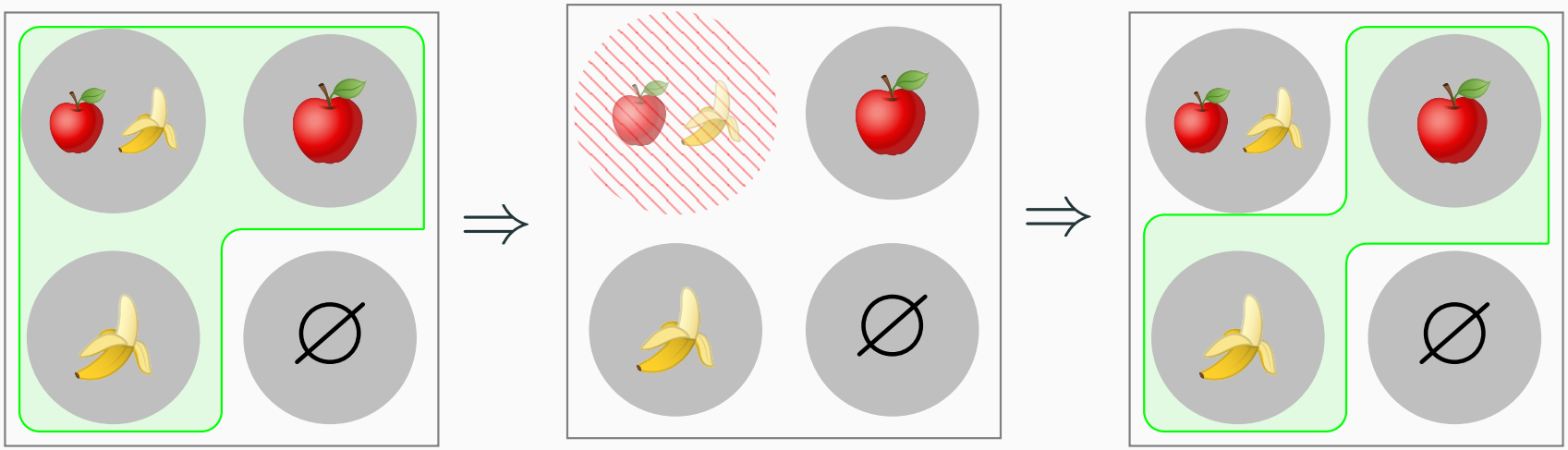
Deriving exclusive disjunction

(2) Ann ate an apple or a banana.
ALT: Ann ate both an apple **and** a banana.
$\rightsquigarrow$ Ann did **not** eat both an apple and a banana.

Exclusive disjunction



Inclusive A OR B | Not (A AND B) | Exclusive A OR B

Acquisition of disjunction

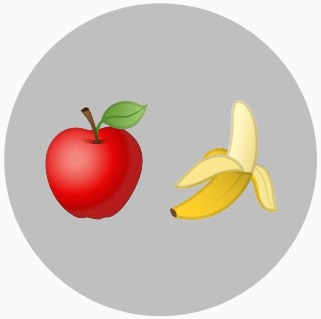
Adults frequently interpret disjunctions exclusively. Inclusive interpretation remains possible (Nicolae et al., 2024).

How do children interpret disjunction?

Childen and alternative-based reasoning

Since Noveck (2001) and Chierchia et al. (2001) a common assumption was that the ability to perform alternative-based reasoning develops late since **children were said to have the inclusive interpretation**.

Experimental evidence:



Adults	✗
Children	✓

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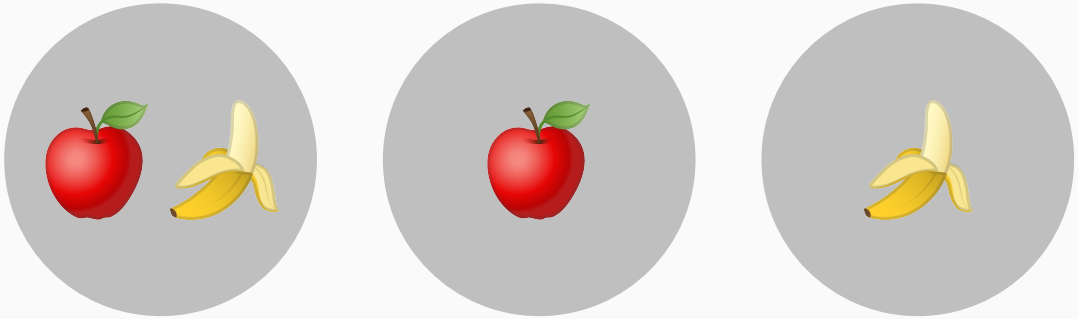

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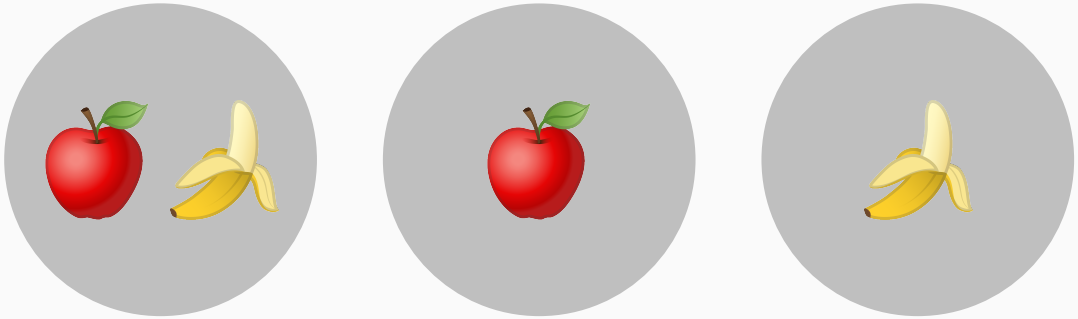


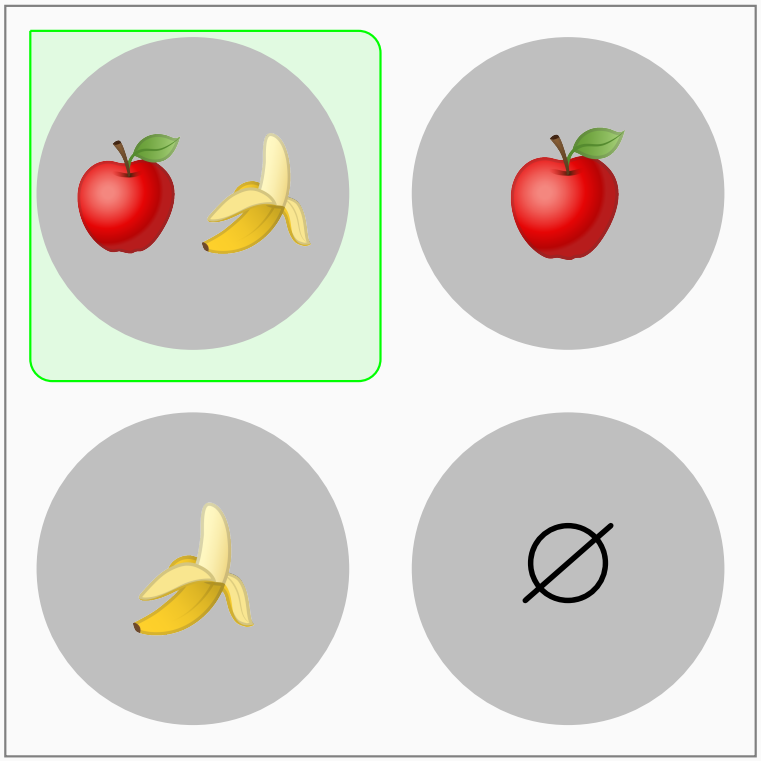
Adults	✗	✓	✓
Children	✓	✗	✗

Singh et al. (2016) as well as Tieu et al. (2017) investigated the remaining cases and found that many children interpret disjunctions conjunctively (cf. Skordos et al. (2020); Huang and Crain (2020)).

Conjunctive readings

(3) Ann ate an apple or a banana.

$\rightsquigarrow$ Ann ate both the apple *and* the banana.



Empirical results (Singh et al., 2016, p.324)

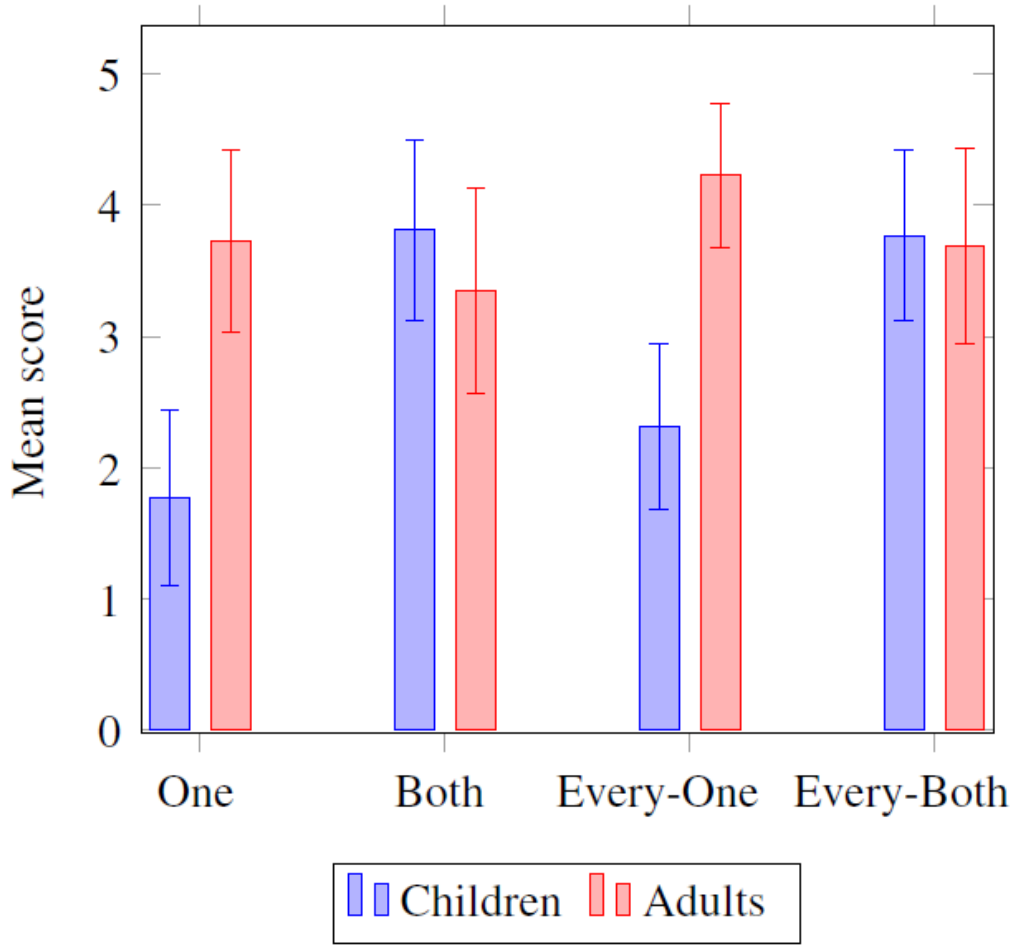


Fig. 3 Comparing children's (n = 31) and adult (n = 26) mean scores on critical conditions (error bars indicate 95 % confidence intervals)

Empirical results (Tieu et al., 2017, p.139)

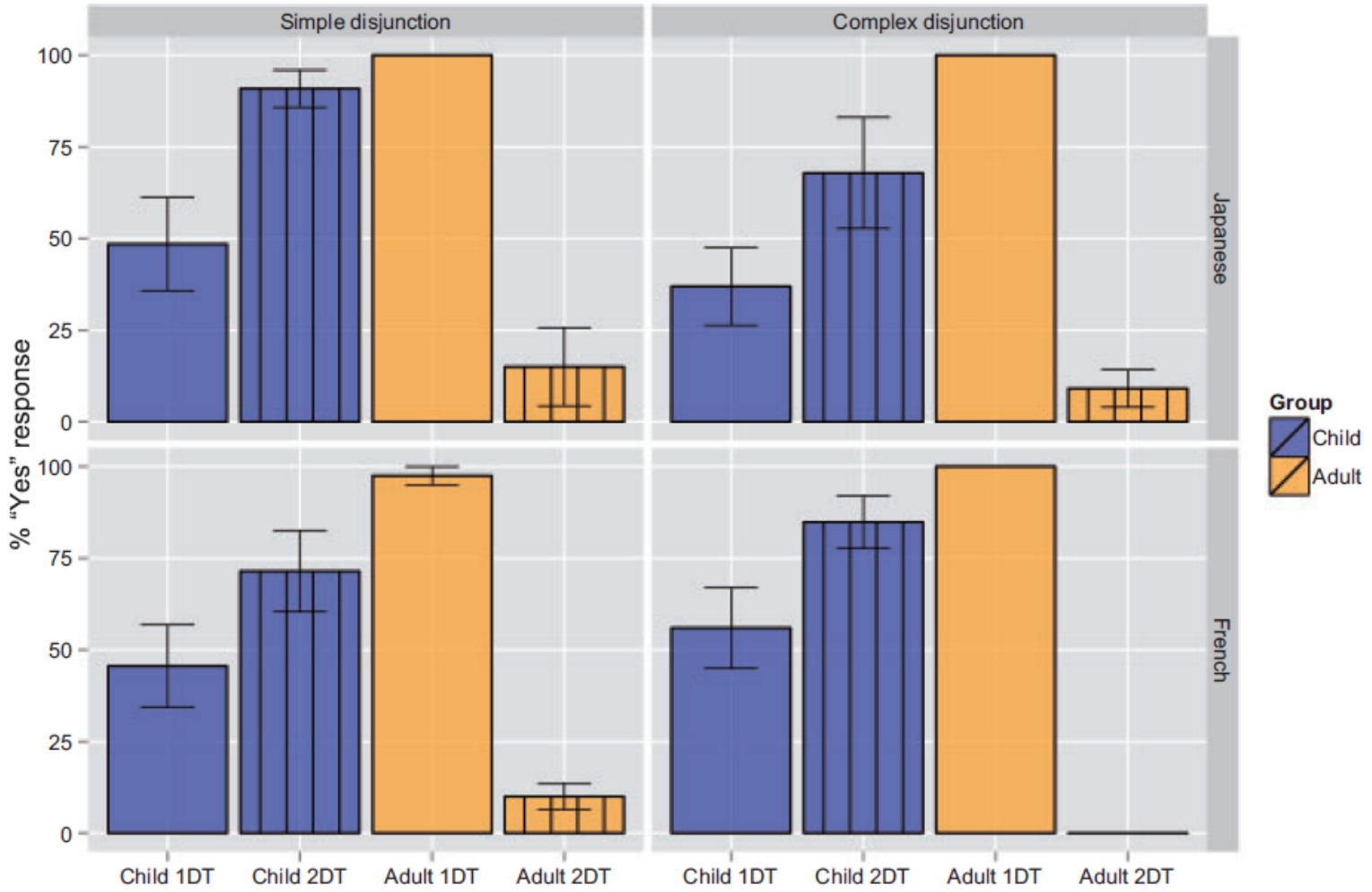
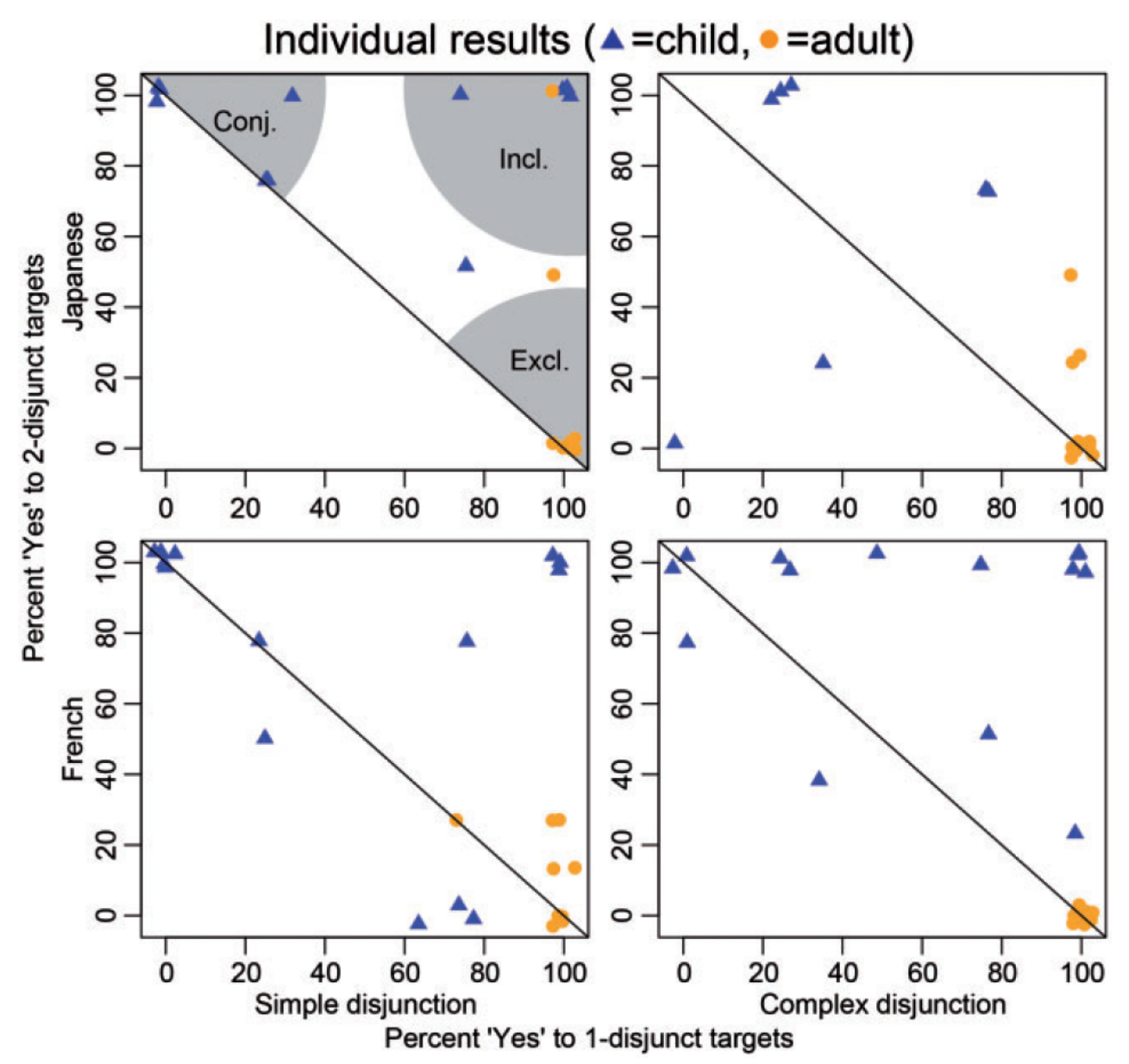


Figure 2 Percentage of *yes*-responses from children and adults to 1DT conditions (plain bars) and 2DT conditions (hashed bars), across disjunction types and languages.

Empirical results (Tieu et al., 2017, p.140)



Why do children interpret disjunction as conjunction?

Null hypothesis

Hypothesis 1 (lexical misanalysis)

Children are genuinely confused between '*or*' and '*and*', as they play the same syntactic role.

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1. Children correctly reason with disjunction in some environments.
(Pagliarini et al., 2018; Su, 2014)

(4) Ann did not eat apples or bananas.
$\not\rightsquigarrow$ Ann did not eat apples and bananas.
$\rightsquigarrow$ Ann did not eat apples and she did not eat bananas.

(5) If Ann conjures up a rabbit, she will get a ball or a star.
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2. Children can be forced to access the disjunctive meaning when evaluating incompatible disjuncts. (Bleotu et al., 2024)

   (6) The squirrel is at the top or at the bottom of the tree.
   $\not\rightsquigarrow$ The squirrel is at the top *and* at the bottom of the tree.

Note on negated conjunctions

Children (and most adults) interpret conjunction under negation as disjunction:

(7) Ann did not eat apples *and* bananas.
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Language	NOT OR	NOT AND	Paper
English	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	(Notley et al.2016)
Japanese	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	(Goro & Akiba 2004; Goro 2007)
Mandarin	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	(Crain 2012, Notley et al.2016)
Turkish	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	(Goro 2007, Geçkin et al. 2016)
Italian	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	(Goro 2007, Geçkin et al. 2016)

Table 1: Children's interpretation of negated conjunction and disjunction.

Alternative-based approach

Proposal by Singh et al. (2016)

Hypothesis 2 (Singh et al., 2016)

Children derive the conjunctive meaning via alternative-based reasoning.

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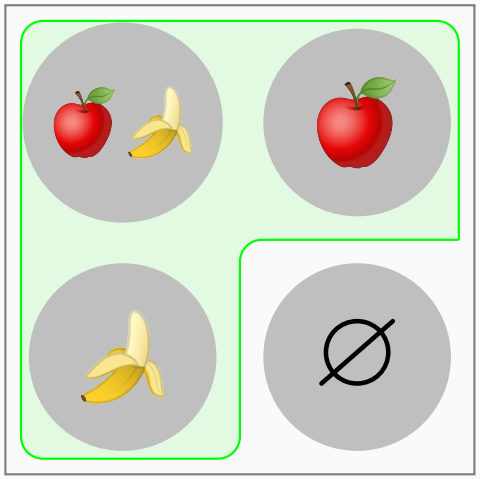
Assumptions:

1. Children can perform (recursive) alternative-based reasoning.
2. Children are not aware that AND is an alternative to OR.
3. Children know the inclusive (logical) meaning of OR, but their alternative-based derivation leads to incorrect results because of (2.).

Alternative-based derivation of conjucntive readings

Step 1 Inclusive OR: Ann ate an apple or a banana.



$\Rightarrow$

Inclusive A OR B

Alternative-based derivation of conjucntive readings

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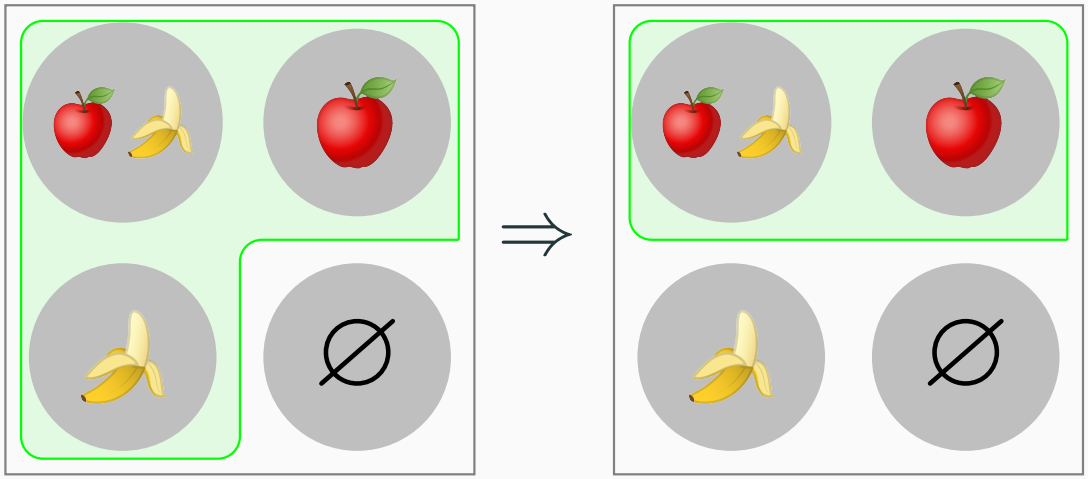
Step 2 *ALT:* 1. Ann ate an apple. 2. Ann ate a banana.



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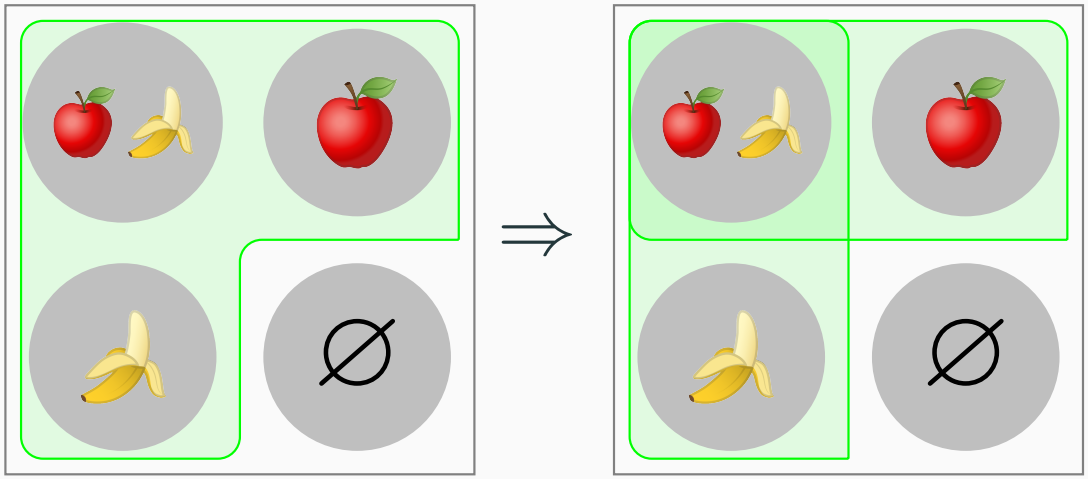


Inclusive A OR B ALT 1

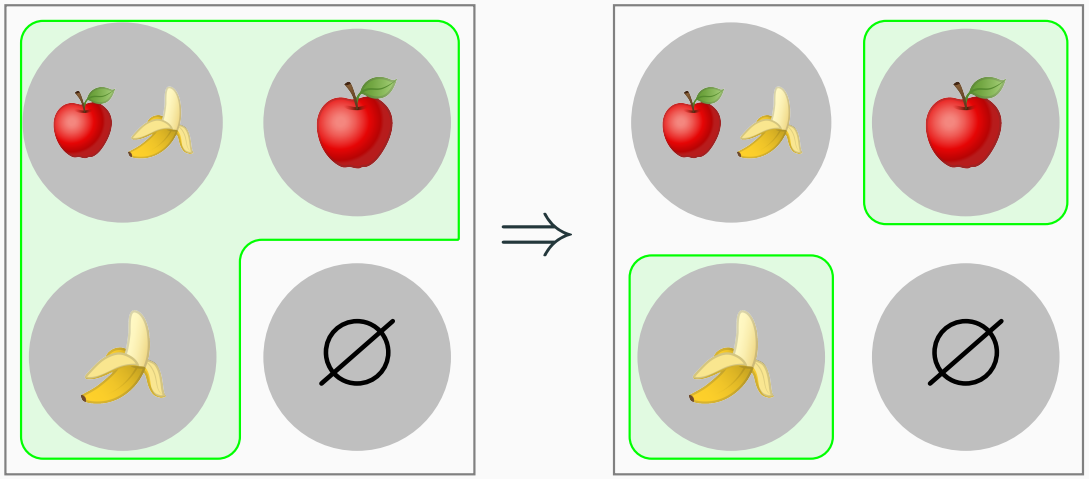
Alternative-based derivation of conjucntive readings

Step 1 Inclusive OR: Ann ate an apple or a banana.

Step 2 *ALT:* 1. Ann ate an apple. 2. Ann ate a banana.

Step 3 Mutual negation of alternatives:

ALT2: 1. Ann ate *only* an apple. 2. Ann ate *only* a banana.



Inclusive A OR B ALT 2

Alternative-based derivation of conjucntive readings

Step 1 Inclusive OR: Ann ate an apple or a banana.

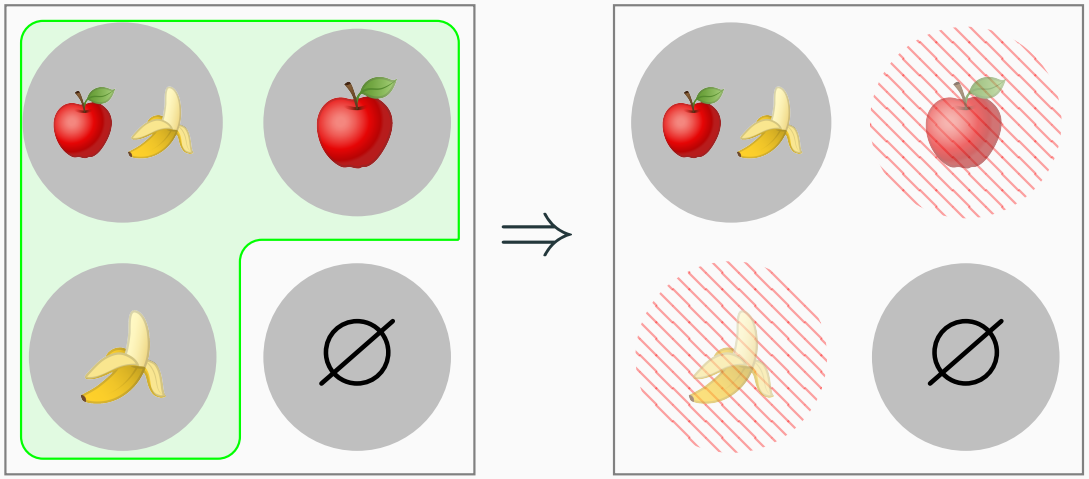
Step 2 *ALT:* 1. Ann ate an apple. 2. Ann ate a banana.

Step 3 Mutual negation of alternatives:

ALT2: 1. Ann ate *only* an apple. 2. Ann ate *only* a banana.

Step 4 Negation of the alternatives:

$\rightsquigarrow$ 1. Ann did **not** *only* eat A. 2. Ann did **not** *only* eat B.



Inclusive A OR B Not ALT 2

Alternative-based derivation of conjucntive readings

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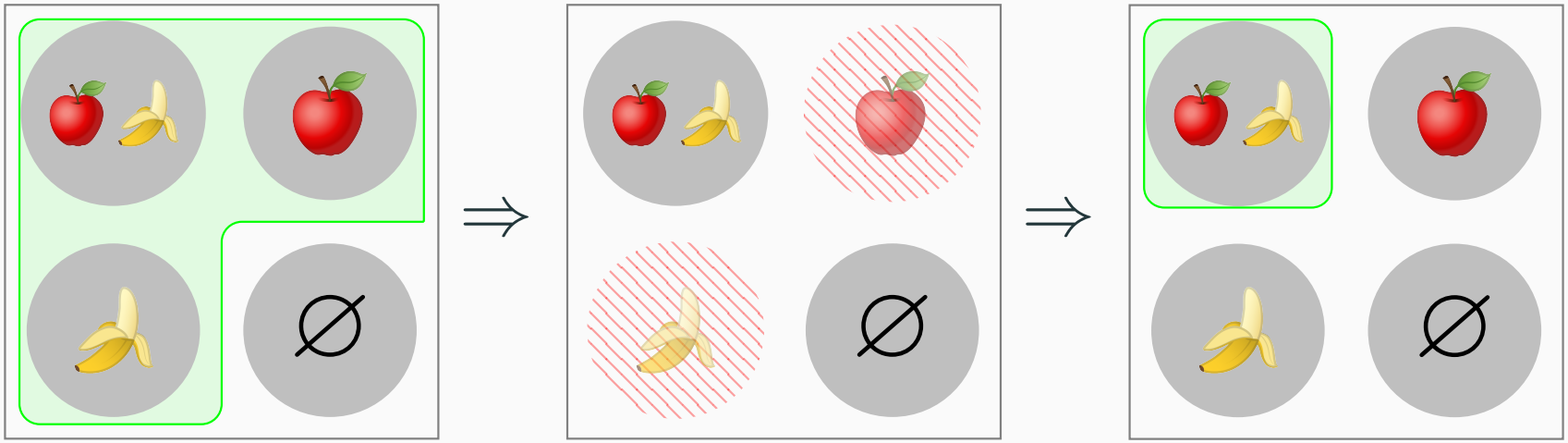
Step 5 Ann ate **both** an apple and a banana.



Inclusive A OR B | Not ALT 2 | A AND B

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Moreover, children systematically and across languages choose that meaning over the inclusive reading.

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Predicted order of acquisition/simplicity:

$$\textsc{inclusive} \precsim \textsc{conjunctive} \precsim \textsc{exclusive}$$

Our proposal

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- Kahneman et al. (1982) observed that human reasoning is systematically affected by **biases**.

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Neglect-zero bias (Aloni, 2022)

Speakers systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*).

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to **empty witness sets**. [Nieder 2016, Bott et al, 2019]

Neglect-zero: illustration

(8) Less than three squares are black.

a. Verifier: [■, □, ■]

b. Falsifier: [■, ■, ■]

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b. Falsifier: [■, ■, ■]

c. Zero-models: [□, □, □] ;[▲, ▲, ▲];

Ignorance inferences of disjunction

Motivation[1]**:**

Ignorance inference **(Grice 1989)**

(9) The prize is in the attic *or* in the garden.
$\rightsquigarrow$ It might be in the attic and it might be in the garden

[1] Neglect-zero bias can account for a range of phenomena involving disjunction, e.g., *free choice* and *distributive inferences*. We will not discuss these applications here.

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Conclusion: in a disjunction, both disjuncts need a (non-empty) witness set of possibilities.

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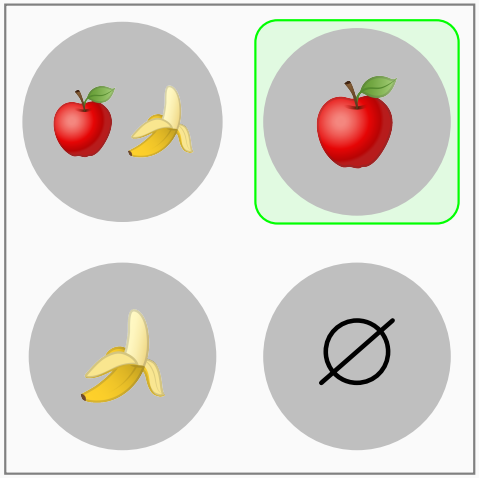
Verifiers of disjunction

(11) Ann ate an apple or a banana.

$\rightsquigarrow$ The speaker does not know which fruit she ate.



Empty verifier

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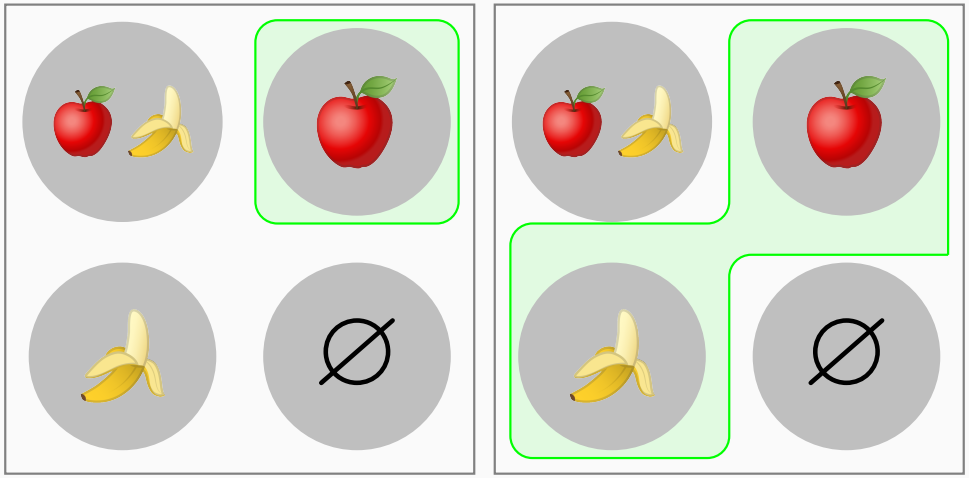

Empty verifier Non Empty verifier

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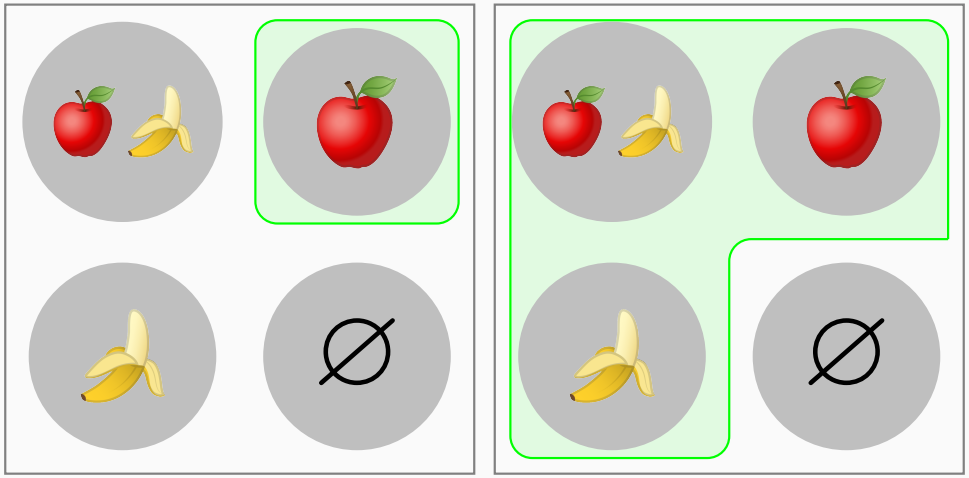

Empty verifier Non Empty verifier

Neglect-zero Aloni (2022)

Illustrations

Ann ate an apple.

- Verifier: [🍎]
- Falsifiers: [🍌], [🍐], [ ]
- Zero-models: none

Ann ate a banana.

- Verifier: [🍌]
- Falsifiers: [🍎], [🍐], [ ]
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Ann ate an apple **and** a banana.

- Verifier: [🍎🍌]
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Ann ate an apple **or** a banana.

- Verifier: ?
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- [🍎]; [🍌] are **zero-models** because they verify the sentence by virtue of an empty witness for one of the disjuncts.
- Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded (**neglect-zero bias**).

Novel hypothesis: no-split

Illustrations

(12) Ann ate an apple **or** a banana.

a. "Split" verifier: [🍎 | 🍌]

b. Conjunctive Verifier: [🍎🍌]

c. Falsifier: [🍐]

d. Zero-models: [🍎]; [🍌]

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Hypothesis 3

Children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, do not have the ability to split.

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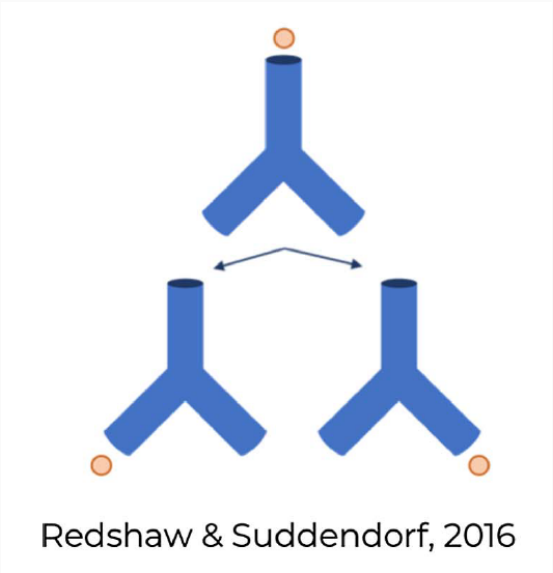
Hypothesis 3

Children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, do not have the ability to split.

- The "split" state in (12-a) involves the entertainment of two alternatives, also a cognitively difficult operation;
- We conjecture that the ability to split states is acquired late.
- The combination of neglect-zero and **no-split bias** can explain non-classical inferences observed in pre-school children.

Additional motivation for the non-split bias

Children have trouble representing multiple possibilities[2].



[2](Beck et al., 2006; Robinson et al., 2006; Redshaw and Suddendorf, 2016; Leahy and Carey, 2020; Phillips and Kratzer, 2024).

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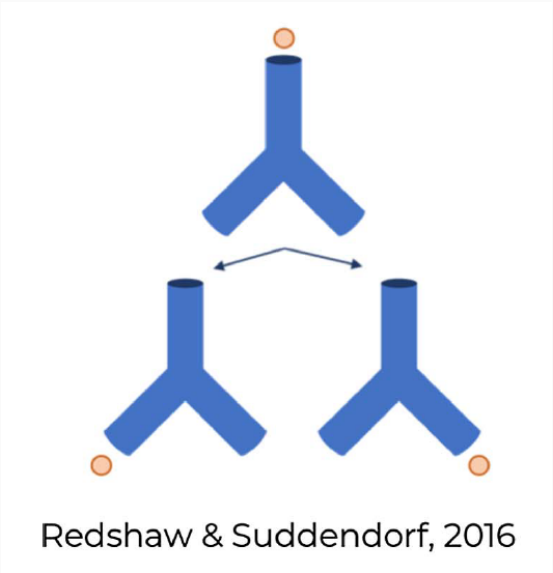
Children have trouble representing multiple possibilities[2].



To cover both exits, one needs to *split* the reality into two possibilities and realise that they can act on both at the same time.

[2](Beck et al., 2006; Robinson et al., 2006; Redshaw and Suddendorf, 2016; Leahy and Carey, 2020; Phillips and Kratzer, 2024).

Our derivation of conjunctive readings

(13) Ann ate an apple **or** a banana.

Deriving ignorance

[🍎] OR [🍌]

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$[🍎]$ OR $[🍌] \overset{NZ}{\Longrightarrow} [🍎] + [🍌]$

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Deriving conjunctive reading

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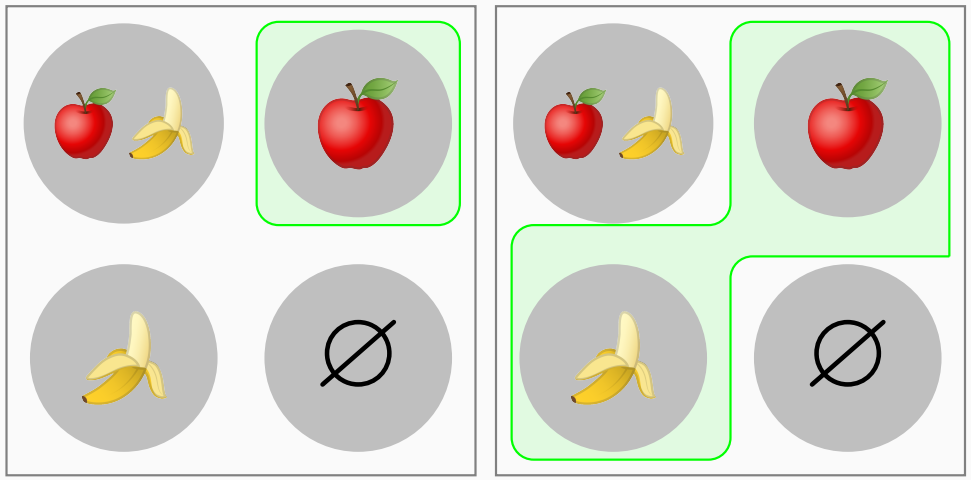
Predicted order of acquisition/simplicity:

$$\text{CONJUNCTIVE} \precsim \text{INCLUSIVE} \precsim \text{EXCLUSIVE}$$

Verifiers of disjunction

(14) Ann ate an apple or a banana.



Empty verifier Non Empty verifier

Verifiers of disjunction

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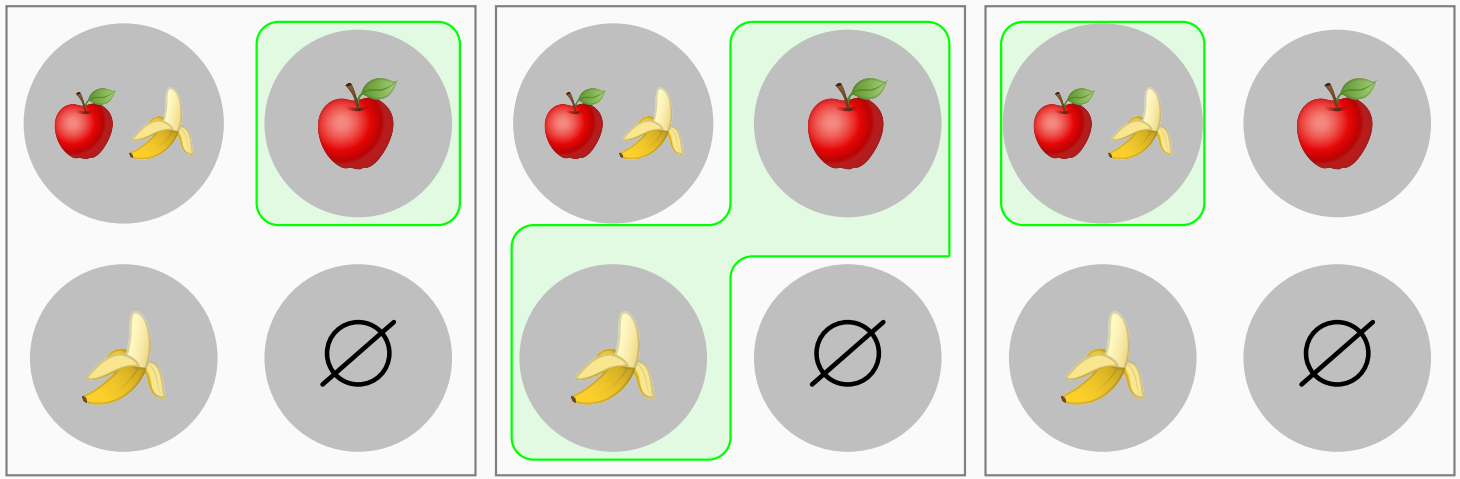
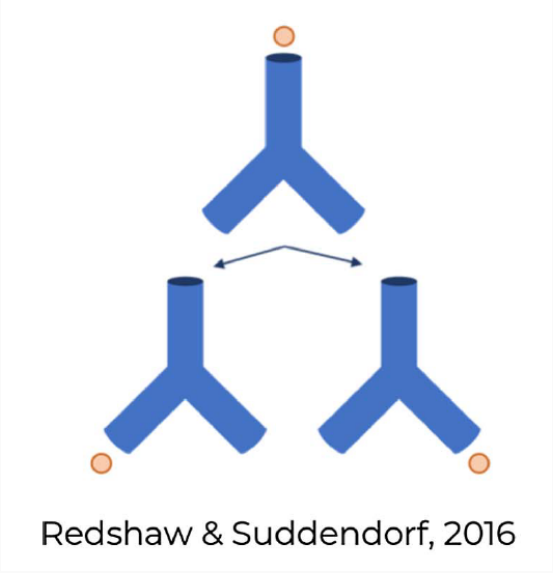


Empty verifier Non Empty verifier No Split verifier

Additional motivation for the non-split bias



(Phillips and Kratzer, 2024)

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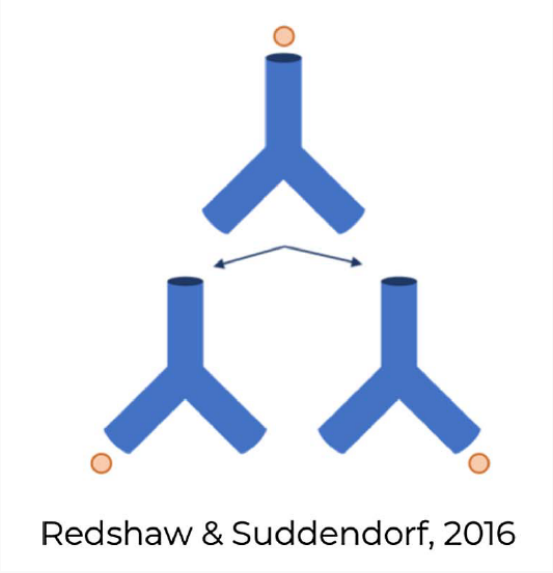

(Phillips and Kratzer, 2024)

Mutually Exclusive Possibilities

[←] OR [→]

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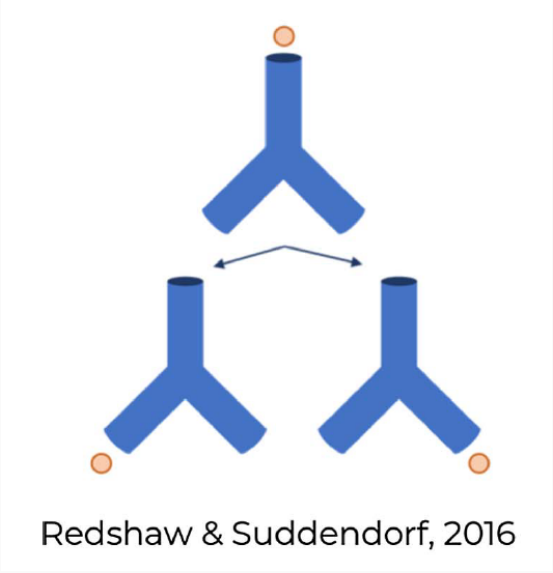
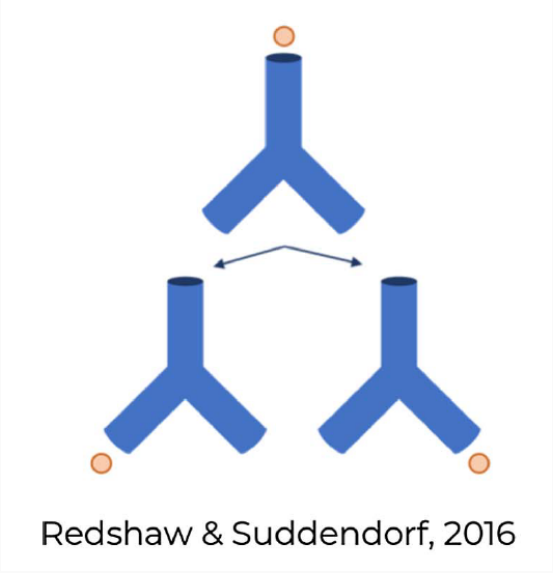


(Phillips and Kratzer, 2024)

Mutually Exclusive Possibilities

$[\leftarrow]$ OR $[\rightarrow] \overset{NZ}{\Longrightarrow} [\leftarrow] + [\rightarrow]$

Additional motivation for the non-split bias



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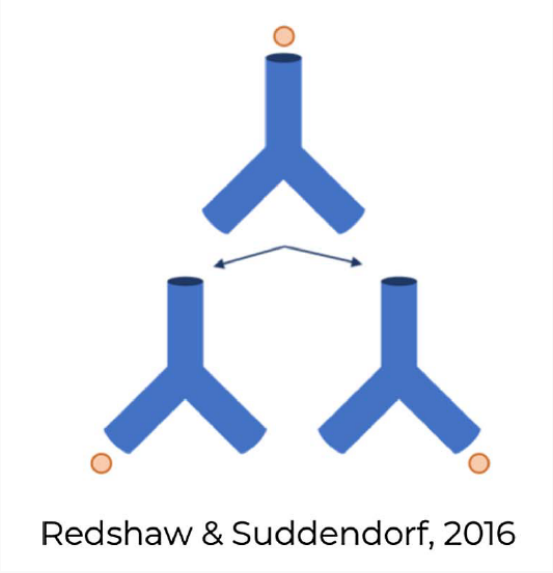
Mutually Exclusive Possibilities

$$[\leftarrow] \text{ OR } [\rightarrow] \overset{NZ}{\Longrightarrow} [\leftarrow] + [\rightarrow] \Longrightarrow [\leftarrow\rightarrow] \#$$

Additional motivation for the non-split bias

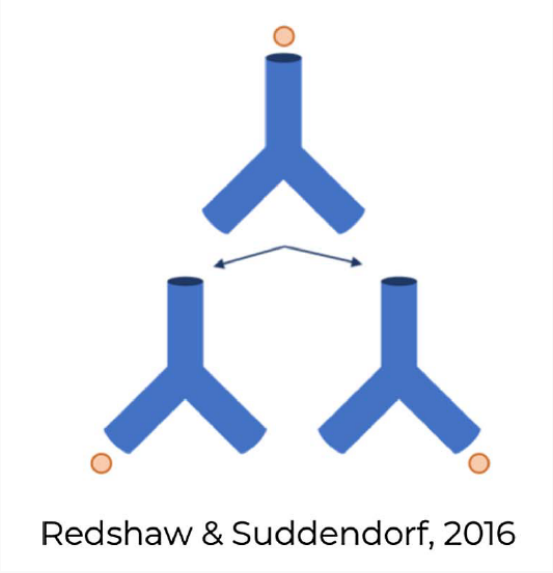


(Phillips and Kratzer, 2024)

Mutually Exclusive Possibilities

$$[\leftarrow] \text{ OR } [\rightarrow] \overset{NZ}{\Longrightarrow} [\leftarrow] + [\rightarrow] \Longrightarrow [\leftarrow\rightarrow] \# \overset{SPLIT}{\Longrightarrow} [\leftarrow \mid \rightarrow]$$

Additional motivation for the non-split bias



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Mutually Exclusive Possibilities

$$[\leftarrow] \text{ OR } [\rightarrow] \overset{NZ}{\Longrightarrow} [\leftarrow] + [\rightarrow] \Longrightarrow [\leftarrow\rightarrow] \# \overset{SING}{\Longrightarrow} [\leftarrow]$$

(Leahy and Carey, 2020)

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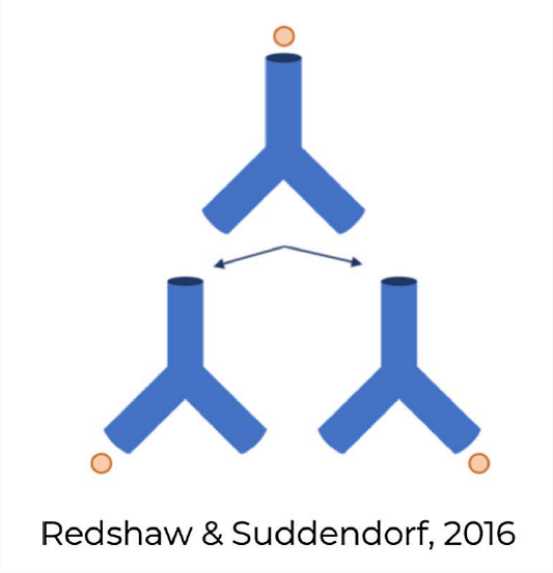

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Mutually Exclusive Possibilities

$$[\leftarrow] \text{ OR } [\rightarrow] \overset{NZ}{\Longrightarrow} [\leftarrow] + [\rightarrow] \Longrightarrow [\leftarrow\rightarrow] \# \overset{SING}{\Longrightarrow} [\leftarrow]$$

(Leahy and Carey, 2020)

(15) The squirrel is at the top or at the bottom of the tree.
$\not\rightsquigarrow$ The squirrel is at the top *and* at the bottom of the tree.

(Bleotu et al., 2024)

Formal framework

Aloni (2022)'s Bilateral state-based modal logic

BSML clauses define logic equivalent to classical modal logic:

$M, s \models p$ iff $\forall w \in s : V(w, p) = 1$
$M, s \mathrel{=\!\!|} p$ iff $\forall w \in s : V(w, p) = 0$

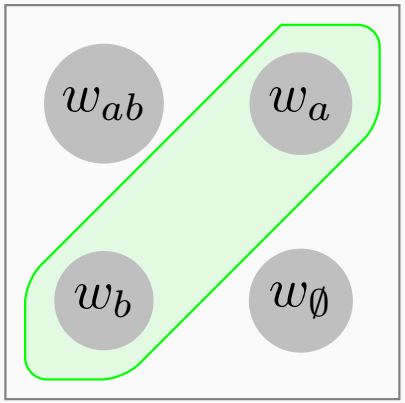
$M, s \models \neg\varphi$ iff $M, s \mathrel{=\!\!|} \varphi$.
$M, s \mathrel{=\!\!|} \neg\varphi$ iff $M, s \models \varphi$.

$M, s \models \varphi \vee \psi$ iff $\exists t, t' : t \cup t' = s \;\&\; M, t \models \varphi \;\&\; M, t' \models \psi$.
$M, s \mathrel{=\!\!|} \varphi \vee \psi$ iff $M, s \mathrel{=\!\!|} \varphi$ and $M, s \mathrel{=\!\!|} \psi$.

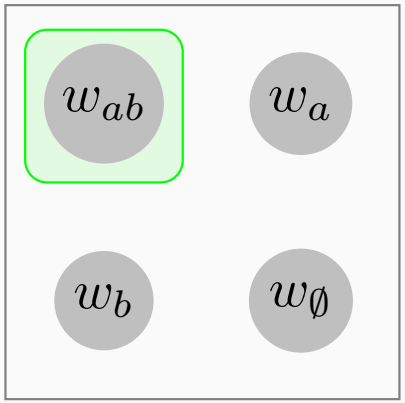
$M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$.
$M, s \mathrel{=\!\!|} \varphi \wedge \psi$ iff $\exists t, t' : t \cup t' = s$ and $M, t \mathrel{=\!\!|} \varphi$ and $M, t' \mathrel{=\!\!|} \psi$.

Disjunction in BSML

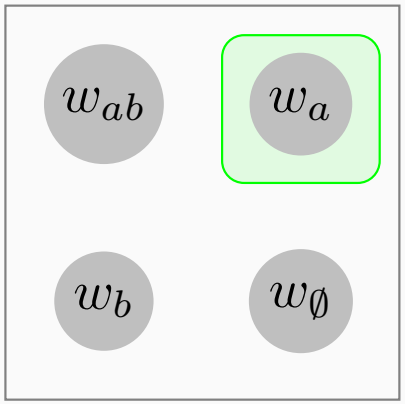
$$M, s \models \varphi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \;\&\; M, t \models \varphi \;\&\; M, t' \models \psi$$



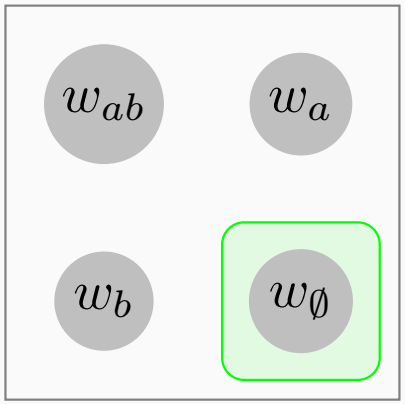
$\models a \vee b$



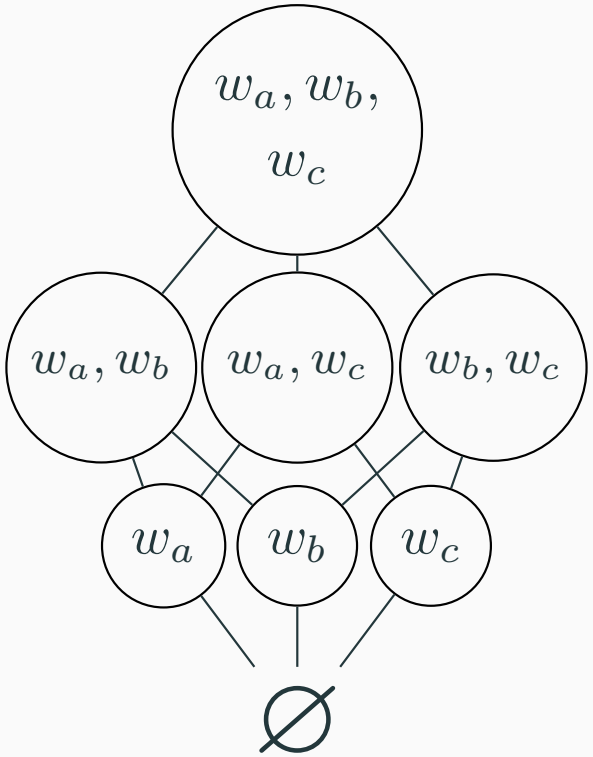
$\models a \vee b$



$\models a \vee b$



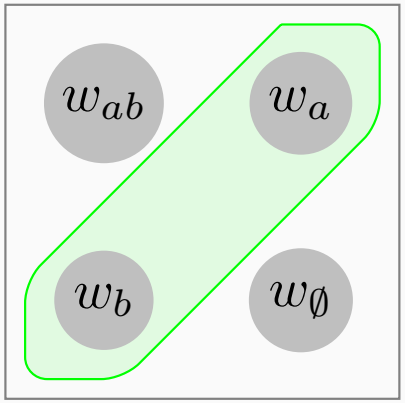
$\not\models a \vee b$



Classical logic

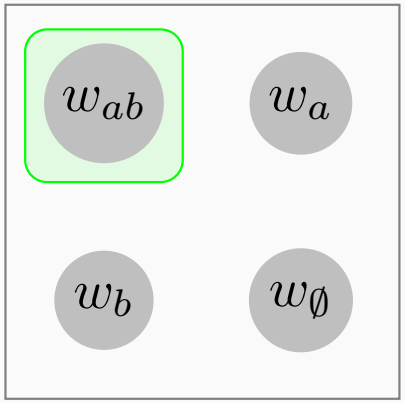
BSML*

Assume that the empty state is unavailable as an evaluation point.[3]



$\models a \vee b$

$\models [a \vee b]^*$



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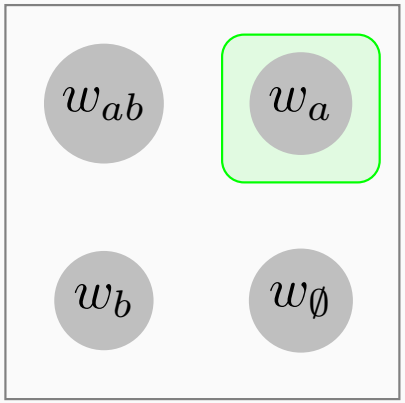

$\models a \vee b$

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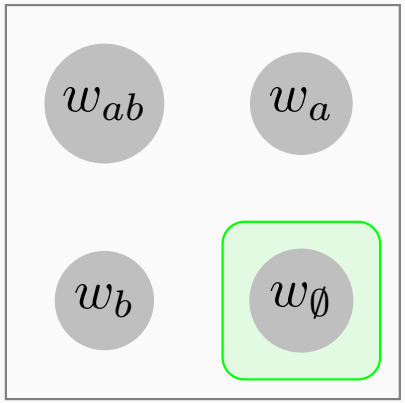
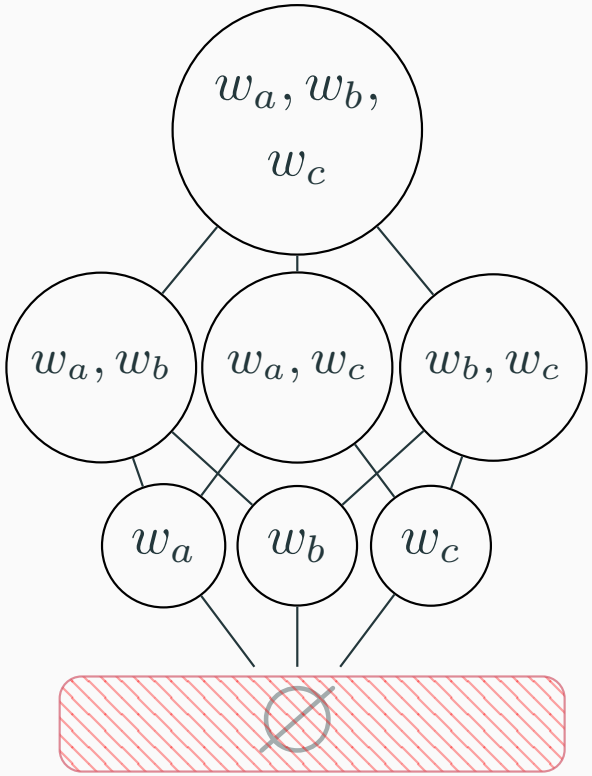


$=\!\!| \; a \vee b$

$=\!\!| \; [a \vee b]^*$

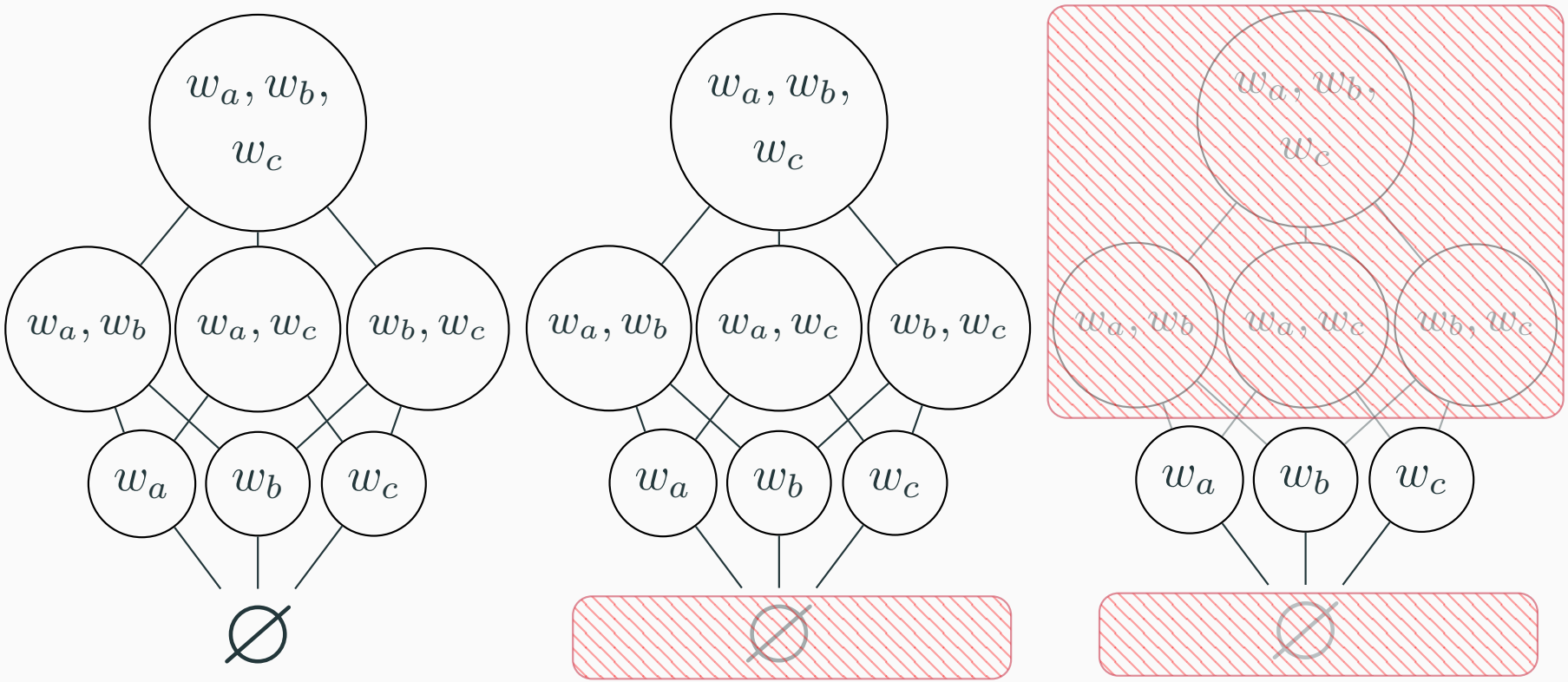


Neglect zero: BSML*

[3] There are crucial differences between BSML$^+$ and BSML* under negation.

BSML^{*ns}

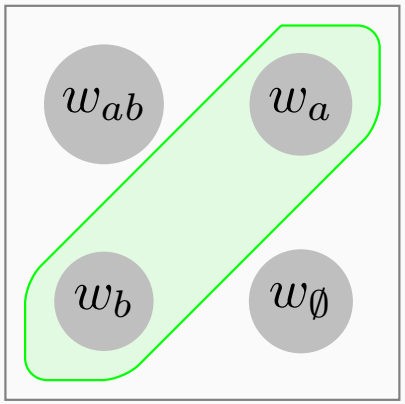
The non-split bias can be modelled by a restriction on the complexity of the available states.[4]



Classical logic | Neglect zero: BSML* | No split: BSML^{NS}

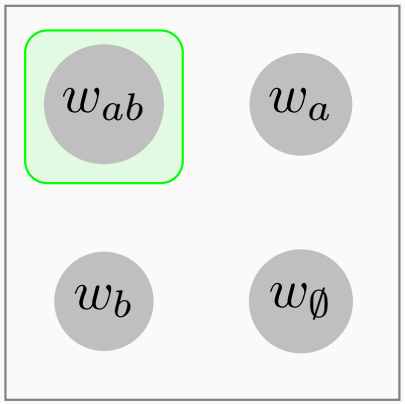
[4] Alternative modelling involves the use of NS atom proposed by Sbardolini (2025) or a flattening operator.

BSML^{*ns}



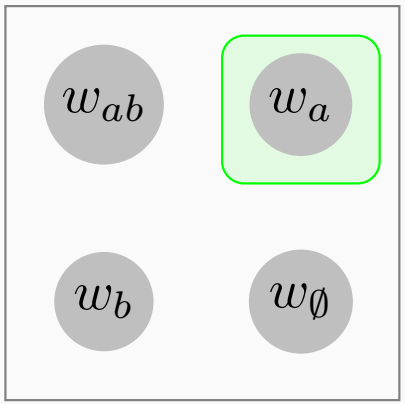
$\models a \vee b$

$\not\models [a \vee b]^{*\mathrm{NS}}$



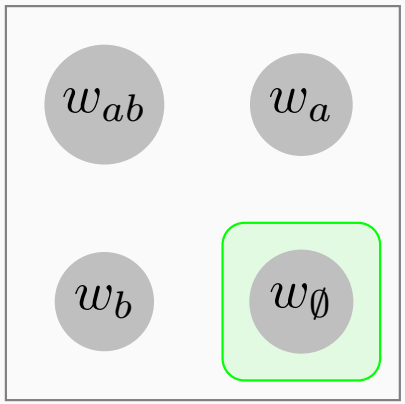
$\models a \vee b$

$\models [a \vee b]^{*\mathrm{NS}}$



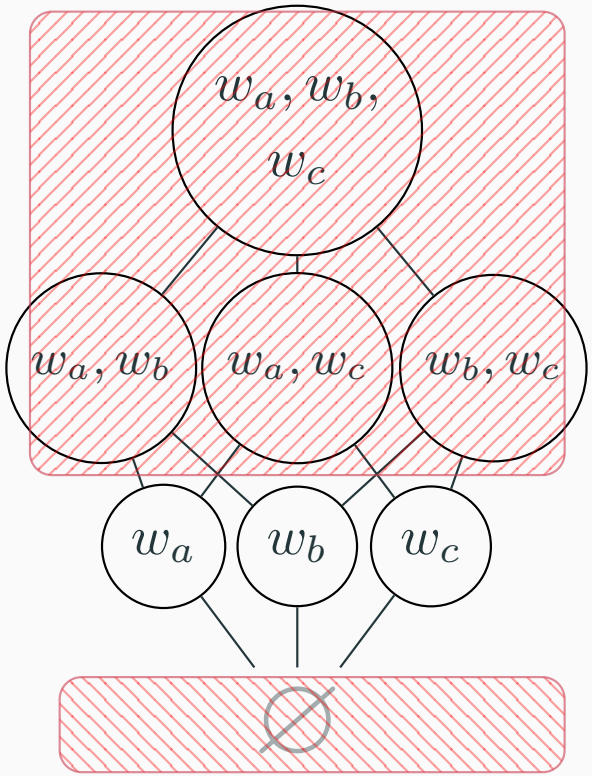
$\models a \vee b$

$\not\models [a \vee b]^{*\mathrm{NS}}$



$=\!\!| \; a \vee b$

$=\!\!| \; [a \vee b]^{*\mathrm{NS}}$



Neglect zero: BSML$^{*\mathrm{NS}}$

Summary of the formal system

- We formally model the biases using Bilateral state-based modal logic (BSML): our theory makes clear predictions: for instance, it predicts equivalence of disjunction and conjunction under negation.

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- BSML offers formal tools to represent the unbiased (literal) and biased (pragmatic) meaning of sentences.
- The biases correspond to model-theoretical restrictions on the complexity of considered states.

Predictions regarding Free Choice inferences

(FC) You are allowed to eat an apple or a banana.
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$\Diamond exh(\alpha \vee \beta) \leftarrow$ Embedded implicature?

Prediction regarding the order of acquisition

Predicted order of acquisition/simplicity:

$$\textsc{conjunctive} \precsim \textsc{inclusive} \precsim \textsc{exclusive}$$

[5] There is a confound in previous studies which did not test if conjunction is accepted in partial-truth situations.

Prediction regarding the order of acquisition

Predicted order of acquisition/simplicity:

$$\textsc{conjunctive} \precsim \textsc{inclusive} \precsim \textsc{exclusive}$$

- Conjunction is children's default interpretation of a connective.

(Bleotu et al. forthcoming in JoS)

- There are more conjunctive children in the younger age groups[5].

(Bleotu et al.)

- Need for a longitudinal study.

[5] There is a confound in previous studies which did not test if conjunction is accepted in partial-truth situations.

Conclusions

1. Children sometimes (but systematically) interpret disjunctions conjunctively.
2. We proposed a cognitive bias approach to explain this phenomenon.
3. Our approach predicts that the conjunctive interpretation is a simplification and should be acquired before the inclusive interpretation.
4. We predict and explain conjunctive free choice, which is difficult to explain for the alternative-based approaches.

Predicted order of acquisition/simplicity:

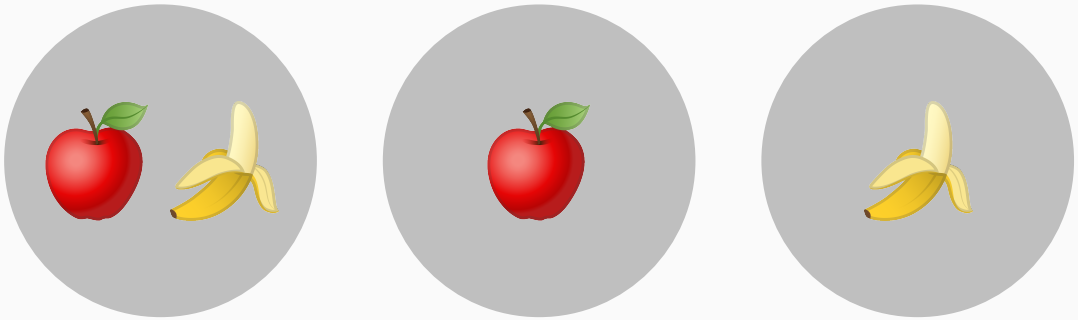
$$\textsc{conjunctive} \precsim \textsc{inclusive} \precsim \textsc{exclusive}$$

Thank you!

Partial-truth children

New work by Bleotu et al. shows that many (especially younger) children exhibit so-called *partial-truth* behaviour. This was not considered in the previous studies and may be a potential confound for the results.

(16) a. Ann ate an apple **or** a banana.
b. Ann ate an apple **and** a banana.



	OR	AND	OR	AND	OR	AND
Exclusive	✗	✓	✓	✗	✓	✗
Inclusive	✓	✓	✓	✗	✓	✗
Conjunctive	✓	✓	✗	✗	✗	✗
Partial-truth	✓	✓	✓	✓	✓	✓

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